DI Physics Hons

Paper-I

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Angle of Twist and Angle of Shear:

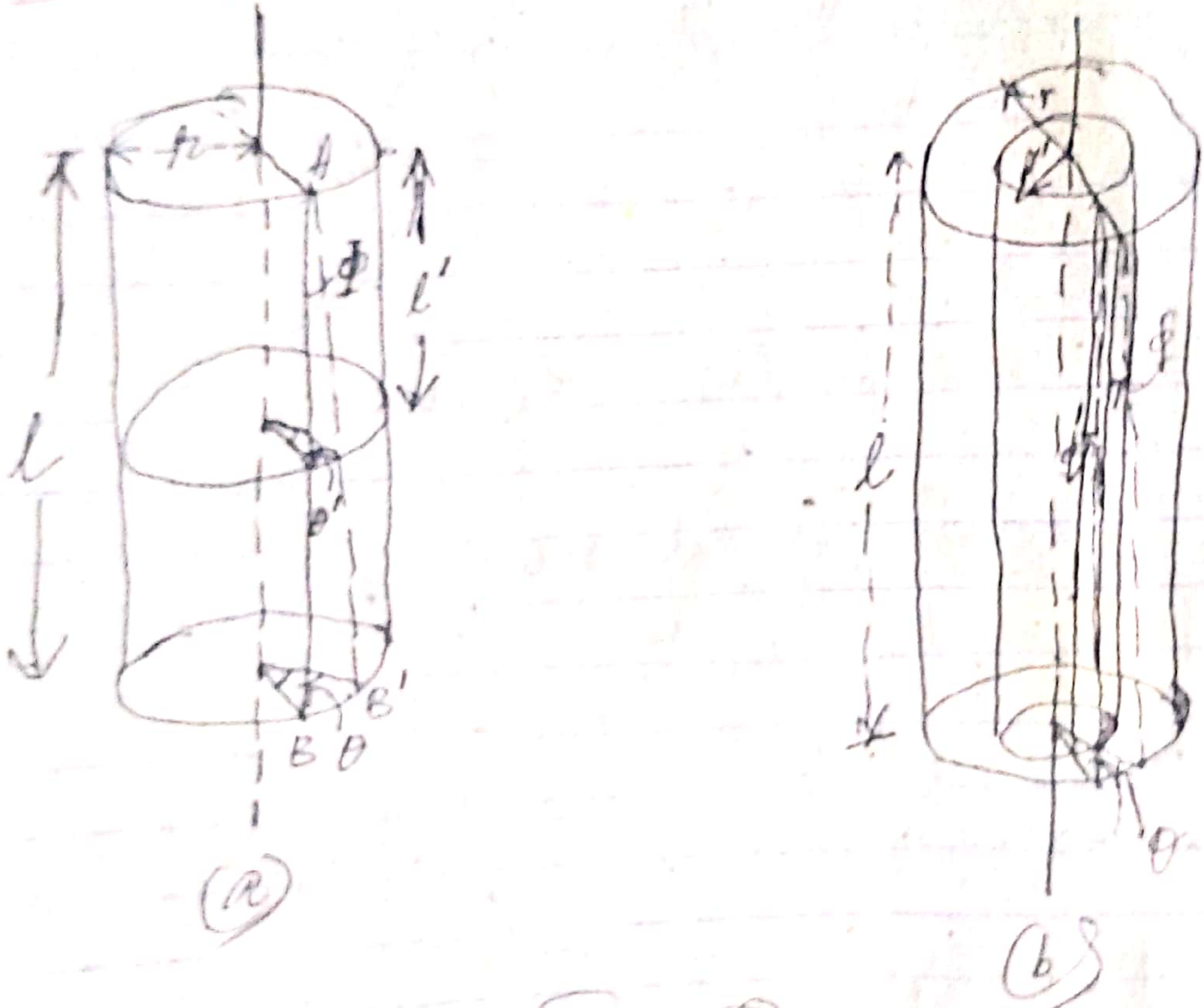

Fig (1)

Consider a cylindrical rod or wire of length l and radius r clamped at the upper end and twisted by a couple at the lower free end [Fig (1)]. By doing so, each circular cross-section of the rod rotates through an angle. This angle is called the angle of twist and is proportional to the distance of cross-section from the clamped end. If the angle of twist at the free end is θ, then clearly

$$r\theta = l\Phi = BB', \quad \text{--- (i)}$$

where Φ is the angle of shear.

From fig. 1 (a) it is clear that θ is constant for the cylindrical surface of radius r.

In any section normal to the axis at a distance, say l', from the fixed end, let the angle of twist be θ', then

$$r\theta' = l'\Phi \quad \text{--- (ii)}$$

Comparing equation (i) & (ii), we see that as $l' < l$, hence $\theta' < \theta$.

Thus we find that the angle of twist, which is a constant in any section normal to the axis at a given length from the fixed end, goes on decreasing as the distance from the fixed end is decreased. At the fixed end, $l = 0$, hence $\theta = 0$.

Fig. 1 (b) shows the shearing of any two concentric cylindrical layer of radius r and r'. The angle of twist θ at the lower free end is a constant for the outer cylindrical surface

$$l\Phi = r\theta$$

and for the inner cylindrical layer

$$l\phi' = r'\theta$$

as $r' < r$, $\therefore \phi' < \phi$.

Hence, we see that the angle of shear, a constant for any cylindrical surface, goes on decreasing as the radius of the surface decreases and that for $r = 0$, $\phi = 0$. It is a maximum on the surface of the cylinder.

Twisting Couple on a Cylindrical Rod or Wire:

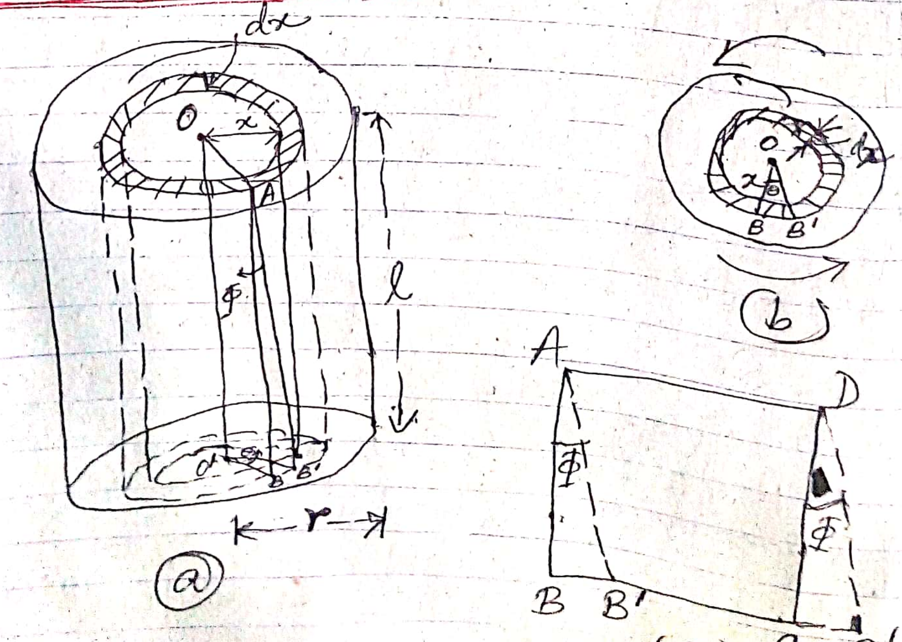

Fig. ①

Consider a cylindrical rod of length l and radius r. The upper end of the rod is kept fixed and a twisting couple is applied at its lower end in a plane perpendicular to its length. Due to elasticity of the material of the rod, a restoring couple is set up in it to resist or oppose the twisting couple. In the position of equilibrium, the two couples balance each other. Couple = ??

Let us imagine the cylinder to consist of a large number of thin coaxial cylindrical shells and consider one such shell of radius x, thickness dx and length l. As the rod is twisted, a line AB (parallel to its axis) on the surface of the cylindrical shell takes up the position AB' through an angle BAB' $= \Phi$. The angle Φ is the angle of shear. For understanding Let this cylindrical shell were cut along AB before twisting and flattened out, it will form the rectangular plate ABCD, but after twisting it acquires the shape of a parallelogram AB'C'D [fig 1(c)].

Clearly, this is a case of shear and angle ϕ is the angle of shear. If θ is the angle of twist at the lower end of the rod, then

$$BB' = AB\,\phi = OB\,\theta$$

or, $$l\phi = x\theta$$

$$\therefore \phi = \frac{x\theta}{l}$$

Let F be the tangential force acting on the base of this thin cylindrical shell, producing a shear. Then

$$\text{tangential stress} = \frac{F}{\text{area of the base of the shell}}$$

But, area of the base of the shell = circumference × thickness

$$= 2\pi x . dx$$

$$\therefore \text{tangential stress} = \frac{F}{2\pi x\, dx}$$

If n is the modulus of rigidity of the material of the shell, then

n = tangential

$$\therefore n = \frac{F}{2\pi x \cdot dx \cdot \phi} = \frac{F}{2\pi x \cdot dx} \cdot \frac{l}{x\theta}.$$

whence

$$F' = \frac{2\pi n\theta}{l} \cdot x^2 dx.$$

The moment of this force about the axis OO' of the rod

$$= \frac{2\pi n\theta \cdot x^2 dx \cdot x}{l} = \frac{2\pi n\theta}{l} \cdot x^3 dx$$

This is equal to the couple required to twist the shell of radius x and thickness dx through an angle θ.

Integrating the above expression between the limits $x = 0$ and $x = r$, we get the total twisting couple, i.e., twisting couple on the rod

$$= \int_0^r \frac{2\pi n\theta}{l} \cdot x^3 dx$$

$$= \frac{2\pi n\theta}{l} \cdot \frac{r^4}{4} = \frac{n\pi r^4}{2l} \cdot \theta$$

If $\theta = 1$, then twisting couple per unit twist $= \frac{n\pi r^4}{2l} = C$ (say).

This C is called the torsional rigidity of the material of the wire. Since twisting couple is numerically equal to the restoring couple, C is also called the restoring couple per unit twist.

If, however, the cylinder is hollow with inner and outer radii r_1 and r_2 respectively, then the couple required to twist the cylinder through θ radian is

$$= \int_{r_1}^{r_2} \frac{2\pi n \theta}{l} x^3 dx$$

$$= \frac{\pi n (r_2^4 - r_1^4)}{2l} \theta .$$

Work done in Twi...